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Toil and Trouble

Near-rational bubbles, rational near-bubbles, pyramid schemes, and overbetting

The idea that asset bubbles can be rational is old but, as far as I know, was formulated precisely only in the late 1970s. Rational bubbles dropped out of favor after 1987, being replaced by behavioral finance explanations. Rationality gained some attention after the Internet bubble burst in 2000, and actually got hot after the housing crash of 2007. As a result of this renewed interest, I decided to dust off some work I did in the mid-1980s on the relation between rational bubbles and Kelly overbetting.

The basic story that bubbles are rational is that every investor rationally expects to sell at a profit to a greater fool, even though each investor (including the greater fool) knows the asset price will eventually crash. At first glance, that seems to violate some kind of value conservation law, but that does not have to be the case. The logic is simplest if we confine ourselves to a zero-interest-rate economy with a single risky asset.

Double or better-than-nothing

Suppose the risky asset has a price process that doubles every period with probability $1/2$ and goes to \$2 otherwise. The expected value of the price next period is the current price plus \$1. Since the asset has a positive expected return, it could be a rational investment every period. Yet we know that the price must eventually go to \$2, so buying it at any higher price and holding it is certain to result in an eventual loss.

It is true that the expected value of this asset increases the farther you go out in the future. If the asset sells for \$16 today, the expected value 100 periods from today is \$116. But that \$116 is composed of a 99.99 percent chance of having less than \$10,000, in which case the asset is worth an average of \$13, and 0.01 percent chance of the asset being worth over \$10,000, in which case its average value is \$843,776. \$16 of the \$116 expected value is contributed by the 10^{-30} chance of getting 100 winning coin flips in a row and having an asset price of over 20 decillion dollars. Thus, the long-term expected value is supported only by infinitesimal chances of gaining astronomical profits, which in practice we know can never be paid. While at any price this asset could be attractive to hold for one period, it is not rational to hold indefinitely at any price.

Although the rationality of buying this

asset is debatable, the rationality of shorting it is not. No one with finite wealth should short this asset. Suppose the asset sells for $16 and you have total wealth of $16,368. If you short it and hold until its price falls to $2, you make $14 1,023 times out of 1,024 and lose $16,368 one time in 1,024. You have an expected loss of $2. You have negative expectation shorting at any price, using any stopping rule consistent with finite wealth.

When looking at this problem in the 1980s, I noticed the similarity to overbetting on an asset with an ordinary price process. Suppose an asset has serially independent daily price movements, up 10 percent on 51 percent of days, and down 10 percent the other 49 percent of the time. The expected return is 0.2 percent per period, which would seem to make it a candidate for rational investment.

The trouble is something called "volatility drag." Every time the asset goes up and down 10% in price, it loses $10\%^2$ or 1 percent (the volatility drag is 1% / 2 = 0.5 percent per day, because it costs 1 percent per two days on average). Over 100 days, if the asset experiences the expected 51 up and 49 down moves, it will have 49 matched up/down moves, costing 1 percent each, plus two unmatched up moves gaining 10 percent each. The result is a loss of 26 percent. The expected return is $1.002^{100} - 1 = 22\%$, but the most likely outcome is a loss of 26 percent. All your expected profit comes from the 2.8 percent of the time in which you have a return of 450 percent or better.

After ten years, on average you will turn $1 into $1.002^{2,600}$ = $180. But there is a 99.4 percent chance that the terminal price will be less than $180, much less, at only $0.92 on average. There is a 0.6 percent chance that you will strike it rich and have an asset with an average price of $31,599.

It's worse than that for a trading strategy as opposed to an asset. The probability is negligible that you will survive ten years.

There is only one chance in 10,000 that you won't have a drawdown of more than 90 percent at some point, and much smaller drawdowns than 90 percent would terminate trading for all real traders. Even the winning paths are very likely to have extreme drawdowns. Conditional on finishing the ten years above the $180 expectation, on average you will have a drawdown of 98 percent before you get there. Your unconditional average over all paths is to have a 99.9 percent drawdown.

Time and volatility drag happeneth to them all

If you generate price paths for this asset or cumulative wealth paths for this strategy, you will see bubble-like behavior. Values will increase rapidly, with increasingly wild swings, until there is a dramatic drawdown to near zero from a dizzying peak. The asset or strategy may recover from some deep drawdowns, but eventually it goes toward zero for good. In retrospect, critics will charge recklessness or foolishness or greed, and supporters will cite extreme bad luck and 20 standard deviation events. But the risk manager knows that the real story is less dramatic. The blow up resulted from the inexorably accumulating volatility drag. "*The race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favor to men of skill; but time and volatility drag happeneth to them all*." That's right, it's not chance.

The reason I call this overbetting is that we can solve the problem by giving the trading strategy five times the capital. Now the gains and losses are 2 percent each, the volatility drag is $2\%^2 / 2 = 0.02\%$, half the expected daily return of 0.04%. This is the optimal Kelly bet. Over ten years, you expect to turn $1 into $1.0004^{2,600}$ = $2.83 instead of $180, an 11 percent annualized return versus 68 percent from the ±10% version, but you will survive to collect the expected return and 19 times out of 20 the ±2% version does better than the ±10% version over ten years. For an asset the equivalent fix is to invest only 20% of your money in the asset, and rebalance daily to keep investment at the 20% level.

Of course, there is nothing magic about increasing some abstraction called capital. We can't put a pile of monopoly money or real money next to the trader and hope to change outcomes.[1] The key is to change the trader's response to gains and losses. After a winning trade, she should increase her positions by 2%, not 10%. In the case of an asset, the holder should take profits after each gain so the net holding increases 2%, and buy more of the asset after each loss so the net holding declines only 2%.[2]

I won't go into the details of the math, but there is a deep connection between rational bubble theory and Kelly overbetting. Rational bubbles require a stochastic process that "explodes" – that is, fails the Feller Explosion Test. Loosely speaking, the volatility increases fast enough with price so that the price is not stable at any finite value. Since the expected value stays finite, the probability distribution of future price becomes an infinitesimal probability of an unboundedly high price and an almost certain loss. The Feller Explosion Test is a function of the expected return divided by the variance of return and the discrete analog is that expected return is less than variance, which is the overbetting region.

The overbetting analysis suggests that we can label the investor in the double-or-$2 bubble price process described above as an irrational overbetter. A rational person will put 100% of wealth in the asset when it is priced at $2, since it can only go up (she would put in more than 100% if that were allowed). At $4, a Kelly investor will use half his wealth to buy the asset. I don't claim that Kelly is the only rational way to invest, but it is a rational strategy. If we can show that the bubble process supports a Kelly strategy, then we can call it rational.

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Sellers but no buyers

Assume that there are 1,000 shares of the $2 asset, and that the economy's total wealth is $2,000. Everyone invests 100% in the asset. When the asset goes to $4, total wealth is $4,000, but investors want to put only half of that in the asset.[3] Unfortunately, there is no one to buy. One possibility is that the asset can be converted directly to consumption goods. In this case, the process is entirely rational. The economy invests its entire $2,000 wealth when the price is $2. When the price goes to $4, the economy takes out $2,000 by converting 500 shares to consumption assets and holds the remaining 500 $4 shares. If the price goes up to $8, total wealth is $6,000 (the $2,000 taken out after the first period, plus 500 $8 shares of the risky asset). Investors will want one-sixth of the $6,000, or $1,000, invested in the risky asset, so 375 shares are liquidated to raise $3,000 in consumption assets, and 125 $8 shares of the risky asset remain.

Each successful flip after the first brings total wealth halfway to $8,000, and the terminal loss will reduce total wealth by one divided by the peak asset price. This is clearly a rational process for the investors, but it's not what we mean by a bubble. Someone observing only the price history would see a bubble, but since the asset prices are convertible into real consumption, these are fundamental value gyrations, not a speculation-driven process.

If the asset cannot be converted to consumption goods at will, then we have to distinguish between value and price. The asset's value will follow the process above, by which I mean that people believe that at some (possibly indefinite) future time the asset can be converted to consumption goods at its value at that time, but the asset has no consumption value today.

People will continue to invest 100% of wealth at a value of $2, because value cannot go down from there, so the price will be $2 as well. But consider what happens if the asset value jumps to $4. At a $4 price, people want to sell half their holdings, but they have no buyers. If a Kelly investor thinks the asset can be converted to consumption goods at its value in the following period, he would pay $3.20 for it rather than its $4 value. He would put half his wealth in the asset at a price of $4, but if he is to be induced to put 100% of his wealth in the asset, the price can be no more than $3.20.

In general, when the value is V, a Kelly investor who believes the asset can be converted to cash at its value next period will pay $4V / (V + 1)$ for it. So, while the value process may be a bubble, the price process is capped at $4. The argument regresses. Suppose the value of the asset is $4 and a Kelly investor thinks the asset can be converted to cash only in two periods at its value at that time ($2, $4, or $16). She knows that if the value goes to $8 next period, the price will go to 4 * $8 / ($8 + $1) = $3.56. Therefore, at a value of $4, the Kelly investor requires a price of $2.56, not $3.20, to put 100% of wealth in the asset. If the conversion to cash is likely only at a remote future date, the Kelly investor will not pay significantly more than $2 for the asset, whatever its value.

Investors may bid up the price of a risky asset without immediate consumption value, but only to a limited extent

Once again, we see rationality, but no bubble. Investors may bid up the price of a risky asset without immediate consumption value, but only to a limited extent and only after considering when and at what price it can be converted to real consumption.

Growth

Another way to try to get a rational bubble out of this process is to assume that total wealth is increasing, even though we only have one risky asset. In the Internet bubble, for example, we could assume that technological advances increased total wealth indirectly, through anticipated future utility from Massively Multiplayer Online Role-Playing Games, seeing funny pictures of drunken acquaintances on Facebook, and getting free music and movies using BitTorrent. In the 1970s commodity bubble (or real asset bubble), it could be that nominal wealth increased through fear of erosion of currency value rather than increased future utility.

The housing bubble fits neither case. If expected future utility increased, we would expect all asset prices to be affected by the bubble, but it was limited to real estate. If the increase was in nominal wealth due to fear of inflation, we cannot explain the simultaneous increase in debt prices (which contributed to or even caused the bubble in real estate). The explanation could be a decline in real interest rates making all future cash flows more valuable, accompanied by a decline in anticipated business profitability so that equity prices were less affected. We could also posit more complicated explanations about anticipated changes in taxes or regulations that might protect nominal gains of homeowners and creditors but not holders of other financial assets. However, my purpose today is not to make an empirical case for what does cause bubbles, but to see if rational bubbles are possible in theory.

To get bubbles, we need more than just exogenous increases in wealth. If there is unlimited opportunity for real investment, increases in wealth will lead to more real investments, not increases in asset prices. People will start new companies rather than bidding up the equity of existing ones. Also the increases must be unexpected, or there must be constraints against investing anticipated future wealth. Expected future wealth is present wealth, and if it can be invested it should affect behavior in the same way as current wealth. While a series of positive wealth surprises followed by a larger negative surprise is possible, I don't consider any associated rise and fall in asset prices to be a rational bubble. It would be a statistically unlikely realization of the random exogenous wealth process.

Following this line, a bubble is a result of financial repression that restricts new real investment and also borrowing against future antici-

pated wealth. These restrictions could result from fundamental factors. There might be limited real investment opportunities available. The anticipated future wealth might be in forms that cannot be exchanged (like expected future enjoyment of technological advances). However, as a practical matter, I suspect any financial repression that causes bubbles would result from regulations, taxes, or restrictions on markets. People are pretty clever about finding profitable ideas to exploit, and figuring out how to exchange future value for current ventures. If that's not happening, it's probably because they're not free to make the attempts.

Exploding wealth

To model this, we can use a simple asset value process and let the wealth process explode. Suppose total wealth begins at \$2 at t = 0, and every period either doubles or goes back to \$2 with equal probability. Again, interest rates are zero and there is one risky asset. In this case, the risky asset can be converted to cash only at some fixed future time, and is either worthless or converts to \$1,024 in cash with equal probabilities, independent of the total wealth process. The total wealth described by the random process does not include the value of the risky asset.

At price P, Kelly investors will want to put $(512 - P) / (1{,}024 - P)$ fraction of their wealth in the risky asset. For equilibrium at wealth W, $P = 512 + W/2 - (512^2 + W^2/4)^{0.5}$, or $P \approx W / 2 - W^2 / 4{,}096$. Most of the time, say at values of W < \$128, the price process will look a lot like the wealth process divided by 2, so it will appear to have bubble characteristics. The price will often soar from its initial \$1 price to \$7.94, \$15.75, and \$31.00 as W goes to \$16, \$32 and \$64, then crash back down to \$1. However, on the rare occasions that W gets higher than those values, the W^2 term kicks in and the price increase is moderate even as W continues to double. P will never go above \$512, even as W increases without bound. \$512 is the asset's expected cash conversion value (and since interest rates are zero, also the expected present value). So the asset price process will look like a bubble almost all the time, but will in fact be fully rational betting in anticipation of real future cash flows.

Talk isn't always cheap

Another stylized fact about bubbles is that bubble investors talk up their investment to get new investors to come in, and to get existing investors to ramp up investments. This is sometimes interpreted as fraudulent, but it needn't be. We could model bubbles as investment opportunities that are not known to all investors at once. As the asset value goes up, existing investors still believe correctly that it has positive expected value, but wish to reduce their personal stakes to avoid overbetting. If the asset cannot be converted to cash, they can accomplish this by recruiting new investors into what they sincerely and correctly believe to be a good investment.

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Suppose we have an economy as before, with zero interest rate and a single risky asset selling for \$2, whose price has a 50 percent chance of doubling each period and a 50 percent chance of returning to \$2. However, this time there is a single share of the asset, owned by one person who has private information as to its value. This initial investor has no cash. Everyone else in the economy has \$2 in cash and no risky asset.

The initial investor is happy to have 100 percent of her wealth in an asset whose price will either double to \$4 or stay at \$2. However, once the asset appreciates to \$4, the initial investor would like to sell half her stake for \$2. She can do this by recruiting two new investors, each to buy a 25 percent stake for \$1, thereby putting half of their wealth in the risky asset.[4] This transaction is rational on both sides.

If the price doubles again to \$8, investors want to hold one-sixth of their wealth in the asset. The initial investor has \$6 wealth (the \$2 she got from selling half her stake after period one, plus half the \$8 asset). She wants to sell \$3 of the asset. The two investors who bought in period one each have \$3 wealth (their \$2 initial endowment, minus the \$1 they paid for their share of the asset, plus a quarter of the \$8 asset). They want to sell \$1.50 of the asset each. Combined, the asset owners need to attract 18 new investors, each to invest \$0.33, to share 75% of the \$8 asset among them. At any $P >$ \$2, you need $(P^2 - 3P + 1) / 2$ total investors to each hold their correct Kelly asset amount.

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Building the pyramids

If this investment can be converted to cash at its price at some point before you run out of investors, all transactions are fully rational. This will look like a bubble; in fact it will be a bubble in all respects except that on very rare occasions it will turn out to be a sound investment for everyone. For example, suppose there are ten million potential investors and the investment pays off in cash if the value gets up to \$8,192. For each \$8,192 cash payout when all investors make money, we will observe one bubble that gets up to \$4,096 before crashing back to \$2, two bubbles that peak at

$2,048, four that peak at $1,024, and so on for a total of 255 bubbles that get to at least $16 before crashing. Although we would be unlikely to distinguish this case empirically from a bubble, it is a rational near-bubble.

If this investment can never be converted to consumption assets, with a sufficiently long run of success, you will run out investors in any finite population. That makes it a pyramid scheme in which the last round of investors before the asset falls to $2 transfer money to the investors who got in on earlier rounds. No rational person should invest if the next round would require more investors than exist, so no one should invest in the penultimate round, so no one in the antepenultimate, and so on back to the first round.

The only way to make this work is to assume an investor population[5] that is growing faster than the number of investors required to fund the asset price without anyone overbetting. This is a true rational bubble, but it is not meaningful. Even if you allow an infinite universe, there is no way to be certain that you can always recruit enough investors. Even a tiny probability of ever running out unravels the entire rational chain back to the first investor. On the other hand, in practical terms, this kind of thing can and often does succeed. However rational it may be for the early investors, at some point irrational investors are necessary. If you knew for certain that no one would ever make a negative expected value investment in a pyramid scheme, in finite universes the idea collapses.

The first message I take away from this exploration is that it's not easy to come up with a completely rational bubble model in a finite universe with zero probability of the bubble asset ever having value and no investor ever overbetting. I'm sure you could put some assumptions around that and prove a result, but someone else could add some kind of friction or complexity or quasi-rationality (like disagreements among investors or utility from owning an asset, even if it never returns cash) to show the contrary.

The reason that borrowing to buy an asset evidencing bubble-like behavior suggests irrationality is that inability to borrow against future wealth is a factor that makes rational near-bubbles more likely

No overbetting is a stronger constraint than most of the literature imposes. The more common constraint is no negative or zero expected value bets. With overbetting allowed, it's easy to produce a bubble – you don't even need an exotic price process.

The second message is that we don't need much irrationality to generate a near-rational bubble. Alternatively, we can have rational near-bubbles in which the bubble asset has a very small probability of ever producing consumption value. The factors that support these near rational bubbles or rational near-bubbles are a price process in which variance increases rapidly enough with price to make short-selling irrational, increasing investor population and wealth, restrictions on deployment of capital in real investing activities, and restrictions on investing anticipated future wealth. Increases in non-exchangeable future wealth are doubly potent as they feed two of the factors.

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Bubble sort

The practical problem is to distinguish near-rational bubbles, which investors should generally avoid and regulators should discourage, from rational near-bubbles which are economically beneficial and may be the only way for the economy to shift out of a local maximum to a better state. We also would like to identify pyramid schemes (which could be either irrational or fraudulently induced bubbles) and attacks of overbetting, both of which can resemble rational bubbles.

No purely theoretical analysis is going to solve this problem, and certainly not one as oversimplified as this one. However, the discussion so far does suggest some non-obvious factors to consider. The first is one familiar to risk managers: estimate the volatility drag. Volatility can usually be estimated with some confidence if the asset in question is liquid, and especially if liquid options trade on it. If variance (volatility squared) is higher than any reasonable expected return, people are overbetting. If there are data on either individual investors or total cash going in and out of the asset, you can measure volatility drag directly. If it is bigger than the realized return on the asset, you again have an overbetting-fueled bubble. Finally, if volatility increases with asset price — or worse with asset price drawdowns — you have particularly virulent overbetting. These are the same tests you apply to any risk-taking activity.

The importance of short-sellers to bubbles was introduced to the literature by Robert Jarrow. A large and vigorous group of investors with substantial short interest in an asset is strong evidence that it is not a bubble asset. If there are short-sellers but they are the people who create the asset or are affiliated with the people who create the asset, you have a pyramid scheme. If there used to be short-sellers but they were driven away by asset price increases, that suggests over-

betting or a near-rational bubble. If there never was any significant short-selling (assuming that it is legal), you might have a rational near-bubble.

If early investors are selling down their stakes and proselytizing for new investment, it is a positive sign, not a negative one. That's what rational investors do with a near-bubble asset. Of course, it's also what dishonest pyramid scheme promoters do (honest but misguided pyramid scheme promoters proselytize but do not sell down stakes). However, the selling and proselytizing makes it less likely that this is a near-rational bubble or case of overbetting.

If new real investment in the bubble asset is large while the price is going up, it suggests a near-rational bubble or overbetting. Real investment should limit the asset price to replacement cost; if it doesn't, something is probably wrong. A rational near-bubble will more likely result when there are restrictions or delays on real investment, or when additional real investment is impossible. In a pyramid scheme, there is no real investment.

Investors borrowing to buy an asset is evidence in favor of an irrational bubble, but not for the obvious reason. Of course, holding asset volatility fixed, borrowing means betting more, and thus could be overbetting. But, for a given level of volatility, a levered low-volatility asset is no more likely to be overbetting than an unlevered high-volatility asset. I think this association between bubbles and leverage is simply a prejudice against leverage. Borrowing does complicate things, because the creditor may be unaware of how much risk the asset buyer is shifting to him. In some cases, the reason for buying the asset may be to borrow the money. Also, leverage may result in forced sales, generating a bubble-like crash in a non-bubble asset. All of these things are potential problems, but not bubble problems.

The reason that borrowing to buy an asset evidencing bubble-like behavior suggests irrationality is that inability to borrow against future wealth is a factor that makes rational near-bubbles more likely. A person with non-marketable future wealth will act in the same way as someone overbetting with current wealth, inducing bubble behavior in assets, but the price movements will be rational from a lifetime perspective. If the asset investors are getting leverage, it removes one support for a rational near-bubble.

Still the same

I worked on the ideas in this paper from 1985 to 1987, thinking mostly of the 1970s real asset bubble. I rewrote it extensively to incorporate things I had learned from other authors and events since then, but I didn't change any of the numerical examples or conclusions. On many topics, my views have changed radically over the past quarter century and some of my papers and presentations from the 1980s should rest in peace. But my views on bubbles have remained pretty much the same, perhaps only deepening somewhat. I take that as evidence that bubbles are not a fad explanation trotted out after every crash to be forgotten in the next boom. I think bubbles are real, and that understanding them is important.

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ENDNOTES

1. Silly as this sounds, there are nominally intelligent people who believe that the soundness of the financial system rests on digging up gold all over the world and reburying it in central bank vaults; this is the capital superstition, in contrast to rational ideas of capital presented in this article.
2. Something similar can be accomplished by forcing risk-takers to pay cash dividends out of successes and come to market or risk managers for more capital after losses, yet many economists and regulators thinking about an entirely different problem believe that preventing financial institutions from paying dividends increases their safety.
3. In general, when the price is P, a Kelly investor will put $1 / (P - 2)$ fraction of wealth in the risky asset.
4. This is not the initial investor's optimal strategy. She would do better to tell everyone about the investment when it was at $2. With more than a few other investors, she could sell her stake for only a little less than its $3 expected value. Therefore, if we wanted to take this approach to argue for rational bubbles, we'd have to add restrictions about the speed or cost of persuading new investors to join.
5. Actually, it is total investor wealth that matters but since we assumed zero interest and only one risky asset, total investor wealth can only grow if new investors enter.